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M. Slemrod has worked on two aspects of multi-dimensional shock **capturing**. The first project has been a multifaceted effort to understand dynamic liquid-vapor interface propagation from a kinetic point of view. Slemrod has modeled the phenomenon via a Boltzmann like cluster dynamics model. Clusters represent groupings of molecules of various cluster sizes which can collide elastically and inelastically. The inelastic collisions can produce coagulation of clusters or fragmentation of a cluster. A fluid made of only small cluster sizes would represent a dilute vapor while one containing very large cluster sizes would be a metastable supersaturated vapor. The model via various scaling limits gives sets of model equations describing vapor flow in various transition regimes. Furthermore, with his co-authors Slemrod has performed numerical experiments modeling vapor to saturated vapor phase change encounted when a dilute vapor encounters a rigid wall.

Slemrod has lectured on this work at the HYP94, Hyperbolic Partial Differential Equations Conference, S.U.N.Y.S.B, Stony Brook, June 12-15, 1994 in an invited lecture. A conference proceeding article will soon be written.

Two papers on this research have been written and accepted for publication. These are

[1] M. Slemrod, M. Grinfeld, A. Qi, I. Stewart, A discrete velocity coagulation- fragmentation model, to appear in Mathematical Methods in the Applied Sciences.

[2] M. Slemrod, A. Qi, Numerical simulations of cluster formation using a discrete velocity kinetic theory of gases.

Slemrod plans to continue this modeling and computation effort. The next steps are to move onto continuous velocities as in the classical Boltzmann equation and solve the continuous velocity model numerically with possible collaboration of a member of the U.W. Computer Sciences Department. The computational issues here are considerable. The hope is to run several workstations in parallel hence avoiding the need for an off campus supercomputer.

The second shock **capturing** effort has been in the study of the spherical piston problem for radially symmetric three dimensional compressible fluid flow. The goal here is to solve the problem via the method of limiting artificial viscosity in the natural similarity variable of the problem $\xi = \frac{r}{t}$, r the radius from the origin at the center of the piston. While the problem is classical (a description may be found in the book of R. Courant and K. O. Friedrichs, Shock Waves) there are some important practical reasons for reconsidering it. These are as follows:

(i) The solution of G. I. Taylor, L. Sedov is based on shock **fitting** via the inviscid equations and hence not amenable to finite difference calculations.

(ii) The solution of Taylor, Sedov need not be robust under viscous perturbations especially as respect to vacuum formation (cavitation) on the piston surface.

Slemrod's self-similar analysis seems to indicate for isentropic gas dynamics with pressure given by $p(\rho) = A\rho^{\gamma}$, $\gamma \geq 1$, the issue of cavitation is real and convergence of the viscous solution to the true inviscid solution only occurs when γ is not too large. Large γ

seems to indicate lack of convergence and hence loss of self-similar motion due to cavitation. The question then is: Is the phenomena real or only mathematical. Surprisingly in a recently released report, Application of finite element strategies to nearly incompressible fluid flow; Univ. of Minnesota, AHPCRC Bulletin, Spring 1994, Vol. 4, No. 2, Dr. Gloria Wren describes research done at the Army Research Laboratory. In laboratory experiments cavitation at the piston surface of a spherical piston seems to play a major role in lack of success with the U.S. Army regenerative liquid propellant gun.

Slemrod now plans to (1) complete his research of the isentropic problem and publish the results, (ii) transfer this information to appropriate U.S. Navy and U.S. Army research personnel, (iii) begin an investigation into the full adiabatic non-isentropic spherically symmetric system.